

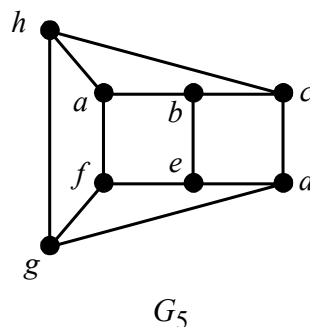
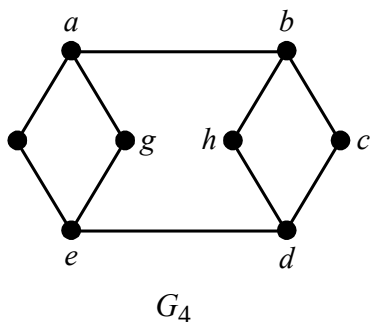
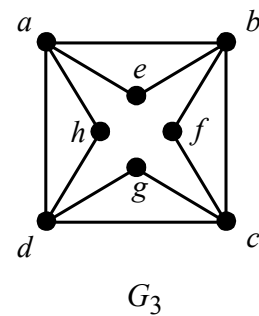
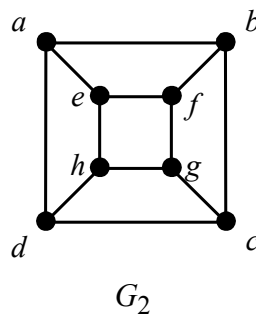
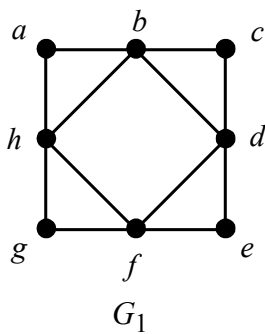
1. Read Chapter 1 of *Wilson*.
2. What is the **adjacency matrix** of a graph  $G$ ?  
How can you tell from the adjacency matrix of  $G$ :
  - (i) the total number of edges of  $G$ ?
  - (ii) whether  $G$  has any loops?
  - (iii) whether  $G$  has any isolated vertices?
  - (iv) the degree of each vertex?
  - (v) whether  $G$  is simple?
3. Describe the adjacency matrix of:
  - (i) the complete graph  $K_n$ .
  - (ii) the complete bipartite graph  $K_{r,s}$ .
4. Draw diagrams to represent each of the graphs whose adjacency matrix is given below.  
Write down the degree of each vertex and state whether the graph is  
(a) simple; (b) regular.

(i) 
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

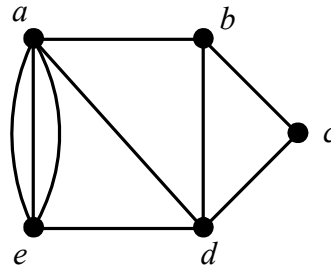
(ii) 
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

(iii) 
$$\begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ 2 & 1 & 2 & 0 & 1 \\ 0 & 2 & 1 & 2 & 1 \\ 2 & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

5. Which of the following pairs of graphs are isomorphic? Justify your answers.



6. What is the **incidence matrix** of a graph  $G$ ?
- What information about  $G$  is provided by (a) the row sums, (b) the column sums of its incidence matrix.
  - Write down the incidence matrix of each of the following graphs.
    - $K_4$
    - $C_6$ , the cycle graph with 6 vertices.
    - the graph  $G$  with the following diagram. (Label the edges first.)



- Draw a diagram to represent the graph whose incidence matrix is:

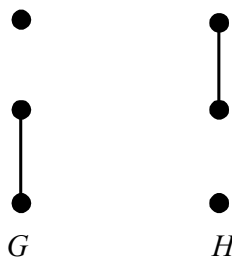
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

- Show that, for  $n \geq 2$ , the complete graph  $K_n$  contains  $K_{n-1}$  as a subgraph.
  - Show that, if  $G$  is a simple graph with  $n$  vertices then  $G$  has at most  $\frac{1}{2}n(n-1)$  edges.

- Let  $G$  and  $H$  be two graphs with no vertices or edges in common. The **union**,  $G \cup H$ , of  $G$  and  $H$  is the graph with vertex set  $V(G \cup H) = V(G) \cup V(H)$  and edge set  $E(G \cup H) = E(G) \cup E(H)$  (where the union of the two edge lists is just one list followed by the other).

The **sum**,  $G + H$ , of  $G$  and  $H$  is obtained by taking the union of the two graphs and then joining each vertex of  $G$  to each vertex of  $H$  by a unique edge.

- Draw a diagram for the sum of the following two graphs.



- What is the sum of two null graphs? (A **null graph** is a graph with no edges.)
- What is the sum of two complete graphs?