MM322: Graphs and Networks

Trees

A **tree** is a connected graph which contains no cycles; a **forest** is a graph which contains no cycles.

Thus a connected forest is a tree; each component of a forest is a tree.

Theorem Let T be a graph with n vertices. The following statements are equivalent (so that, if any one statement is true for T then all the statements are true for T and, conversely, if any one statement is false for T then all the statements are false for T).

(i) T is a tree.

(ii) T is connected and has $n-1$ edges.

(iii) T is connected and every edge is a bridge.

(iv) Given any pair of distinct vertices in T, there is a unique path joining them.

(v) T contains no cycles, but adding any additional edge creates a cycle.

Proof: Wilson, page 44.

Let G be a connected graph. A **spanning tree** in G is a subgraph T which is a tree and which contains every vertex of G.

Theorem Every connected graph contains a spanning tree.

Proof: Let G be a connected graph.

If G does not contain a cycle then G is a tree and so is its own spanning tree.

If G does contain a cycle C then remove one of the edges of C. The resulting graph G' is still connected.

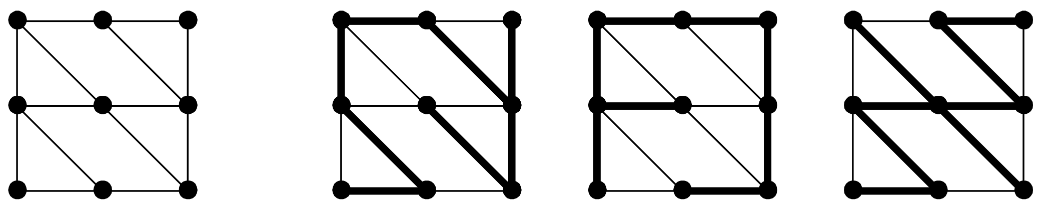
If G' is a tree then it is a spanning tree for G.

If G' is not a tree then choose a cycle C' in G' and remove one of the edges of C' to produce G'' which is still connected.

Continue in this way until a connected subgraph T is obtained which has no cycles; T is a spanning tree for G.

Example

In general, a connected graph G may have many spanning trees. Some spanning trees for a specific graph G are indicated by the following diagrams.



Graph Some spanning trees